rectangular and circular geometry and examined their relationship to bilinear expansions for Green's functions attendant to familiar Sturm–Liouville boundary value problems. Other coordinate systems and cross sections give rise to identities involving Mathieu functions, confluent hypergeometric functions, and so on. These topics are the subject of further investigation.

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Self-Consistent Finite/Infinite Element Scheme for Unbounded Guided Wave Problems

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***Abstract* — An efficient finite-element approach for the eigenmode analysis of unbounded guided wave problems is described using decay-type infinite elements. To determine an optimum set of decay parameters, two algorithms based on successive approximation are presented and their validity is checked via the application to an optical fiber problem.**

I. Introduction

It is well recognized that difficulty is frequently encountered when one wants to solve unbounded field problems using finite elements. To overcome this difficulty, these unbounded domains have in the past been dealt with in various ways, all of which have strengths and weaknesses. To date the main methods in guided wave problems have been simple truncation [1]–[4], the use of analytical far-field solutions [5], the decay-type infinite element approach [6], [7], the exterior finite element approach [8], and the conformal mapping technique [9]. The simplest technique among them is undoubtedly the simple truncation, in which the unbounded domain is truncated to a finite size. However, this technique involves a very large number of nodal points when the field extends farther away from the guiding region. Among other methods the decay-type infinite element approach, in which a finite element is extended to infinity, is often simple and economical and has now been applied successfully to a wide range of problems [10], [11]. A pending question in applying this method is the determination of unknown parameters involved which represent decaying behavior in a far-field region. Although almost all of the authors to date have mentioned this problem, no systematical algorithm for determining the decay parameters has yet been developed [6], [7], [10], [11].

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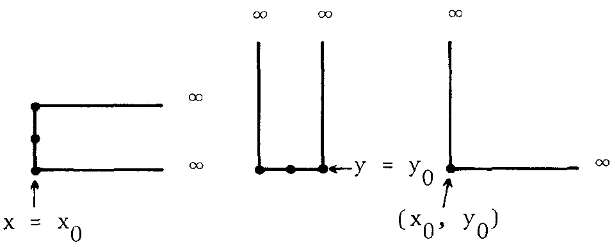

Fig. 1. Infinite elements.

In this paper, a self-consistent finite/infinite element scheme that can be used for the eigenmode analysis of unbounded dielectric waveguide problems is developed. To determine the decay parameters involved, two algorithms based on successive approximation are proposed and their validity is examined by means of the application to an optical fiber problem. By using these algorithms, an optimum set of decay parameters is readily obtainable in a self-consistent iterative way.

II. Determination of an Optimum Set of Decay Parameters

Consider strip-like infinite elements shown in Fig. 1 and expand the field ϕ in each element as

$$\phi = \{N\}^T \{\phi\}_e \qquad (T\text{: transposition}) \tag{1}$$

where $\{N\}$ is the shape function vector of the infinite elements and $\{\phi\}_e$ is the nodal vector for each element.

As a trial function for semi-infinite directions, we choose the following decay function:

$$f(\xi; c) = \exp\{-c(\xi - \xi_0)^p\} \qquad (c > 0, p > 0.5) \tag{2}$$

where c is the unknown decay parameter and $(\xi, \xi_0, c) = (x, x_0, \alpha_x), (y, y_0, \alpha_y)$. If p is set to unity, (2) is reduced to the exponential function [6], [7], [10], [11]; we choose $p = 1$ in the following description.

To determine systematically the best value of c, we propose here the following two algorithms:

A. A Method Utilizing the Field Profile in a Finite Element Region

Fig. 2 shows a schematic illustration of a field profile on the axes. We approximate the field ϕ near the points x_0, y_0 as

$$\phi(x, 0) = u_0 \exp\{-\alpha_x(x - x_0)\} \tag{3}$$

$$\phi(0, y) = v_0 \exp\{-\alpha_y(y - y_0)\}. \tag{4}$$

If we choose other points (x_1, u_1) and (y_1, v_1) corresponding to the nodes in a finite element region, the unknown parameters

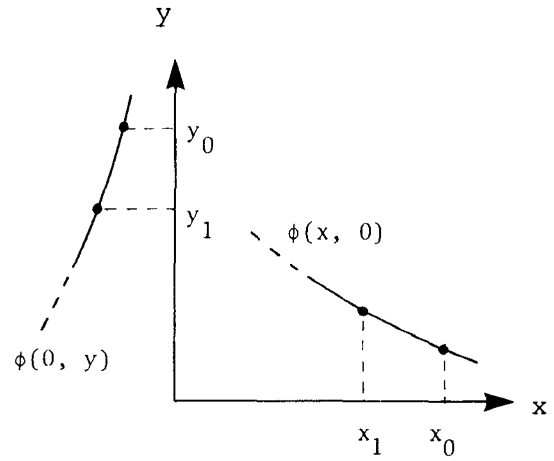

Fig. 2 Schematic illustration of field profile on axes. The region $x \leqslant x_0$, $y \leqslant y_0$ is divided into finite elements, while the region $x \geqslant x_0$, $y \geqslant y_0$ is divided into infinite elements shown in Fig. 1.

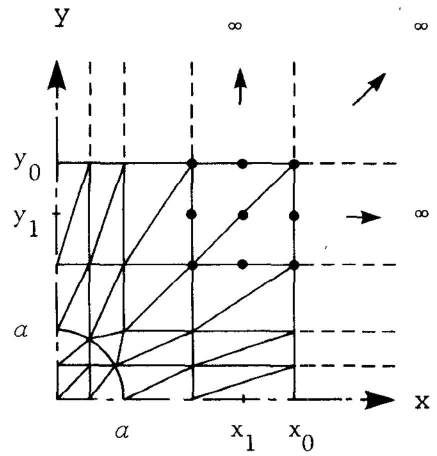

Fig. 3. Element division for optical fiber. The solid and broken meshes correspond to the finite and infinite elements, respectively.

α_x, α_y are obtained from (3) and (4):

$$\alpha_x = \frac{\ln|u_1/u_0|}{x_0 - x_1}, \qquad \alpha_y = \frac{\ln|v_1/v_0|}{y_0 - y_1}. \tag{5}$$

An optimum set of the parameters can be derived self-consistently via the following iterative scheme:

(i) Assign initial values to α_x and α_y in an arbitrary way.
(ii) Solve the matrix equation to obtain u_0, u_1, v_0, v_1 as an eigenvector.
(iii) Calculate α_x and α_y according to (5).
(iv) Iterate the above procedures (ii) and (iii) until the solution converges within required accuracy.

B. *A Method Using the Transverse Wavenumber in a Cladding Region*

The procedure described herein is simpler than that described above and is easy to manage without knowledge of the field profile. In this procedure the decay parameters α_x, α_y are obtained using the transverse wavenumber in a cladding region:

$$\alpha_x = \alpha_y = \sqrt{\left(\beta^2 - n_{cl}^2 k_0^2\right)} \equiv \alpha \tag{6}$$

where β is the phase constant of the waveguide, n_{cl} the refractive index in the cladding, and k_0 the free-space wavenumber. Also

TABLE I
EFFECTIVE INDEX OF FUNDAMENTAL MODE

Number of iterations	β/k_0 for $\beta a = 6$ Algorithm (a) $(\bar{\alpha}_x, \bar{\alpha}_y)$	Algorithm (b) $(\bar{\alpha})$
1	1.46041(0.052, 0.052)	1.46029(0.012)
2	1.46042(0.050, 0.050)	1.46041(0.020)
3	1.46042(0.050, 0.050)	1.46044(0.024)
4	1.46042(0.050, 0.050)	1.46044(0.024)

Simple truncation: 1.45837
Exact calculation: 1.46018

$\bar{\alpha}_x = \alpha_x/\beta$, $\bar{\alpha}_y = \alpha_y/\beta$, $\bar{\alpha} = \alpha/\beta$

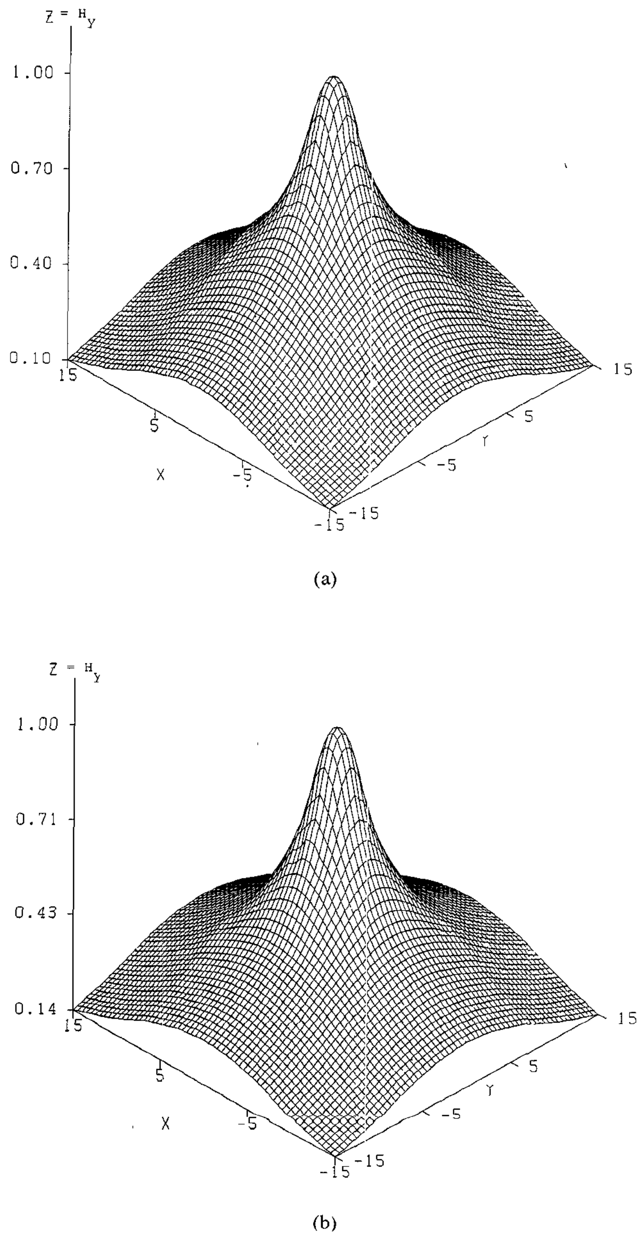

Fig. 4 Magnetic-field distribution in cross section; number of iterations = 1. (a) Algorithm A. (b) Algorithm B

in this case, an optimum value of the parameter α can be determined self-consistently via the following iterative scheme:

(i) Assign an initial value to α in an arbitrary way.
(ii) Solve the matrix equation to obtain k_0^2 as an eigenvalue (note that β is given as an input datum).
(iii) Calculate α according to (6).
(iv) Iterate the above procedures (ii) and (iii) until the solution converges within required accuracy.

Although this method includes only one decay parameter, it does not require the calculation of the eigenvector.

III. Numerical Example

To demonstrate the power of the present algorithms, we consider a round optical fiber since its exact solution is readily available. Making use of symmetry nature, we divide only one quarter of the cross section into quadratic finite and infinite elements, as illustrated in Fig. 3. As a finite element scheme, we use the scalar formulation [2], [3].

Table I exhibits an example of the analyzed results, where the index difference $\Delta = 1$ percent, $n_{cl} = 1.46$, and $\beta a = 6$ (a: core radius), and the initial set of the parameters is $\alpha_x/\beta = \alpha_y/\beta = 0.1$. Note that $\beta a = 6$ corresponds to a case very near cutoff. It is readily found from the table that sufficiently accurate solutions are obtainable only with one time of iteration. On the other hand, the solution for the simple truncation, in which the boundaries $x = x_0$, $y = y_0$ are assumed to be perfect conducting walls and no infinite elements are added to them, is far less accurate than that for the present algorithms.

Fig. 4 displays an example of the field distributions in the cross section. In this figure the region $|X| \geqslant 7$, $|Y| \geqslant 7$ corresponds to that divided into infinite elements. It is seen from the figure that the interface between finite-element and infinite-element regions is smooth in spite of only one time of iteration.

IV. Conclusions

A self-consistent finite-element approach for the eigenmode analysis of unbounded waveguides has been proposed using decay-type infinite elements. Two algorithms have been described for the determination of the unknown decay parameters. Through the application to the eigenmode analysis of an optical fiber, the power of this approach has been successfully demonstrated.

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Analysis of Coupled Microslab™ Lines

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Abstract —**Symmetrically coupled Microslab lines are analyzed with a mode-matching method to build design charts for the propagation constant and characteristic impedance. Results are provided for GaAs/alumina Microslab implementations.**

I. Introduction

Microslab is a novel low-loss quasi-planar waveguide intended for use at millimeter-wave frequencies [1]. The single-line implementation has been studied and the results appear in [2], where a design procedure is presented which minimizes conductor loss, and design charts are given for implementation on GaAs substrates. This paper extends that work by analyzing the symmetrical coupled-line Microslab configuration. Design charts are provided for GaAs/alumina implementations to complement the results in [2]. The design charts to complete the GaAs implementation for insulating layer dielectric constants of 8.2 and 11.5 are not included due to the lack of space.

II. Analysis

The analysis method used to build the design charts is the mode-matching method. The particular procedure is based on the one used in [2]. The method is outlined below to provide the additional details necessary for the coupled-line implementation.

The symmetrically coupled Microslab is shown in Fig. 1. The metallizations are perfectly conducting with zero thickness, and the dielectrics are lossless. A cover plate is added to the structure to discretize the eigenvalue spectrum [3]. Since the strips have equal widths, the structure can be divided along the plane of symmetry with a magnetic (electric) wall to eliminate the odd (even) modes. The divided structure is further subdivided into four regions as shown in Fig. 2 for modal expansion. Extra dielectrics are added to the left and right of the strip in regions 1 and 4 to facilitate checking the program.

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